

REMARKS/ARGUMENTS

Reconsideration of the application is requested.

Claims 1 and 3-9 remain in the application. Claim 1 has been amended. Claim 2 has been cancelled.

In the section entitled "Claim Rejections - 35 USC § 112" on page 2 of the above-identified Office action, claims 1 and 3-9 have been rejected as being indefinite under 35 U.S.C. § 112, second paragraph.

More specifically, the Examiner has stated that it is not clear what kind of state the word "states" in claim 1 refer to.

In band terminology, "states" refer to energy levels, which, for example, can be taken by electrons or holes. Energy bands, as used in band terminology, include "allowed" energy levels and band gaps as well as "forbidden" energy levels. States of the allowed energy levels within the forbidden band gaps represent an exception in semiconductors. The energy levels within the band gaps are shown in Fig. 5 (by broken lines) and described on page 13, line 18 to page 14, line 2 of the specification of the instant application. An excerpt from the book "Lectures on the Electrical Properties of Materials"

(L. Solymar and D. Walsh, 4<sup>th</sup> ed., Oxford University Press, 1990, pages 101-107 and 154-159) is enclosed herewith as a reference to support the above discussion and to facilitate understanding of the concept. Relevant paragraphs are marked at the right margin.

Although the concept "states" is common and well known to a person skilled in the art, the language of claim 1 has been amended to even more clearly define the invention of the instant application.

It is accordingly believed that the claims meet the requirements of 35 U.S.C. § 112, second paragraph. Should the Examiner find any further objectionable items, counsel would appreciate a telephone call during which the matter may be resolved. The above-noted changes to the claims are provided solely for cosmetic and/or clarificatory reasons. The changes are neither provided for overcoming the prior art nor do they narrow the scope of the claims for any reason related to the statutory requirements for a patent.

In the penultimate paragraph on page 2 of the above-mentioned Office action, claims 1, 4-6, and 9 have been rejected as being unpatentable over Kitagawa et al. (US Pat. No. 5,198,690) under 35 U.S.C. § 103(a).

As will be explained below, it is believed that the claims were patentable over the cited art in their original form and the claims have, therefore, not been amended to overcome the references.

Before discussing the prior art in detail, it is believed that a brief review of the invention as claimed, would be helpful.

Claim 1 calls for, inter alia:

said first semiconductor layer including states of allowed energy levels in said first band gap.

As already discussed in detail in the response to the previous Office action dated January 30, 2003, none of the cited references discloses a material with states of allowed energy levels in the band gap, which is formed as a re-emission layer that is exposed to radiation by light of the first wavelength, emits light of a second wavelength, and emits light of mixed colors together with the light of the first wavelength within a single component, as recited in claim 1 of the instant application.

Claim 1 is, therefore, believed to be patentable over the art and since claims 4-6 and 9 are dependent on claim 1, they are believed to be patentable as well.

In the penultimate paragraph on page 4 of the above-mentioned Office action, claim 3 has been rejected as being unpatentable over Kitagawa et al. in view of Henry et al. (US Pat. No. 4,570,172) under 35 U.S.C. § 103(a).

As discussed above, claim 1 is believed to be patentable over the art. Since claim 3 is dependent on claim 1, it is believed to be patentable as well.

In the second paragraph on page 5 of the above-mentioned Office action, claim 7 has been rejected as being unpatentable over Kitagawa et al. in view of Ishikawa et al. (US Pat. No. 5,488,233) under 35 U.S.C. § 103(a).

As discussed above, claim 1 is believed to be patentable over the art. Since claim 7 is dependent on claim 1, it is believed to be patentable as well.

In the fifth paragraph on page 5 of the above-mentioned Office action, claim 8 has been rejected as being unpatentable over Kitagawa et al. in view of Liau et al. (US Pat. No. 4,784,722) under 35 U.S.C. § 103(a).

As discussed above, claim 1 is believed to be patentable over the art. Since claim 8 is dependent on claim 1, it is believed to be patentable as well.

In view of the foregoing, reconsideration and allowance of claims 1 and 3-9 are solicited.

In the event the Examiner should still find any of the claims to be unpatentable, counsel would appreciate a telephone call so that, if possible, patentable language can be worked out.

Petition for extension is herewith made. The extension fee for response within a period of one month pursuant to Section 1.136(a) in the amount of \$110.00 in accordance with Section 1.17 is enclosed herewith. Please charge any other fees which might be due with respect to Sections 1.16 and 1.17 to the Deposit Account of Lerner and Greenberg, P.A., No. 12-1099.

Respectfully submitted,



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# **Lectures on the Electrical Properties of Materials**

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## Preface to the fourth edition

This edition reflects our attempts trying to keep pace with new developments both in materials and in their applications. Recognizing that Optics has now become a subject of great importance in engineering we have expanded the chapter on Lasers, and added a new chapter on Optoelectronic Devices. Some of the revolutionary new results in the field of Superconductors are described in Section 14.9 added at the proof stage, but a full appraisal, we regret, will have to wait for the fifth edition. For stimulating further the interest of students we have increased the number of examples and, for the first time, a *Solutions Manual* has now also become available.

Again, we wish to thank colleagues and pupils for pointing out mistakes and suggesting alterations. In particular we thank Professor Ashok of the University of Pennsylvania who kindly allowed us to include here some of the examples he set for his students. For the pretty picture showing the short-range order of logs in a Canadian river we are indebted to Dr E. P. Raynes of the Royal Signal and Radar Establishment. For Fig. 14.10, providing the explanation for the discovery of high critical temperature oxide superconductors, we are grateful to Professor Müller of the IBM Laboratories, Zürich.

Oxford

December 1987

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electrons inside the metal (more correctly the valence electrons which occupy the outer ring) are entirely free to roam around but they are not allowed to leave the metal.

You may recognize that the model is nothing else but the potential well we met before. There we obtained the solution for the one-dimensional case in the following form:

$$\begin{aligned} E &= \frac{\hbar^2 k^2}{2m} \\ &= \frac{\hbar^2 n^2}{8mL^2} \end{aligned} \quad (6.1)$$

If we imagine a cube of side  $L$  containing the electrons then we get for the energy in the same manner

$$\begin{aligned} E &= \frac{\hbar^2}{2m} (k_x^2 + k_y^2 + k_z^2) \\ &= \frac{\hbar^2}{8mL^2} (n_x^2 + n_y^2 + n_z^2), \end{aligned} \quad (6.2)$$

where  $n_x, n_y, n_z$  are integers.

## 6.2 The density of states and the Fermi-Dirac distribution

The allowed energy, according to eqn (6.2) is an integral multiple of  $\hbar^2/8mL^2$ . For a volume of  $10^{-6} \text{ m}^3$  this unit of energy is

$$E_{\text{unit}} = \frac{(6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 10^{-4}} = 0.6 \times 10^{-33} \text{ J} = 3.74 \times 10^{-15} \text{ eV}. \quad (6.3)$$

This is the energy difference between the first and second levels, but since the squares of the integers are involved the difference between neighbouring energy levels increases at higher energies. Let us anticipate the result obtained in the next section and take for the maximum energy  $E = 3 \text{ eV}$ , which is a typical figure. Taking  $n_x^2 = n_y^2 = n_z^2$  this maximum energy corresponds to a value of  $n_x \approx 1.64 \times 10^7$ . Now an energy level just below the maximum energy can be obtained by taking the integers  $n_x - 1, n_y, n_z$ . We get for the energy difference

$$\Delta E \approx 1.22 \times 10^{-7} \text{ eV}, \quad (6.4)$$

that is, even at the highest energy the difference between neighbouring energy levels is as small as  $10^{-7} \text{ eV}$ . We can therefore say that in a macroscopically small energy interval  $dE$  there are still many discrete

energy levels. So we can introduce the concept of density of states, which will simplify our calculations considerably.

The next question we ask is how many states are there between the energy levels  $E$  and  $E + dE$ . It is convenient to introduce for this purpose the new variable  $n$  with the relationship

$$n^2 = n_x^2 + n_y^2 + n_z^2. \quad (6.5)$$

Thus  $n$  represents a vector to a point  $n_x, n_y, n_z$  in three-dimensional space. In this space every integer specifies a state, that is a unit cube contains exactly one state. Hence the number of states in any volume is just equal to the numerical value of the volume. Thus in a sphere of radius  $n$  the number of states is

$$\frac{4\pi^3 n^3}{3}. \quad (6.6)$$

Since  $n$  and  $E$  are related this is equivalent to saying that the number of states having energies less than  $E$  is

$$\frac{4\pi^3 n^3}{3} = \frac{4\pi}{3} K^{3/2} E^{3/2} \quad \text{with} \quad K = \frac{8mL^2}{\hbar^2}. \quad (6.7)$$

Similarly, the number of states having energies less than  $E + dE$  is

$$\frac{4\pi}{3} K^{3/2} (E + dE)^{3/2}. \quad (6.8)$$

So the number of states having energies between  $E$  and  $E + dE$  is equal to

$$\begin{aligned} Z(E) dE &= \frac{4\pi}{3} K^{3/2} \{ (E + dE)^{3/2} - E^{3/2} \} \\ &= 2\pi K^{3/2} E^{1/2} dE. \end{aligned} \quad (6.9)$$

This is not the end yet. We have to note that only positive values of  $n_x, n_y, n_z$  are permissible; therefore we have to divide by a factor 8. Allowing further for the two values of spin we have to multiply by a factor 2. We get finally

$$Z(E) dE = C E^{1/2} dE \quad \text{with} \quad C = 4\pi L^3 (2m)^{3/2} / \hbar^3. \quad (6.10)$$

Equation (6.10) gives us the number of states but we would also like to know the number of *occupied* states, that is, the number of states that contain electrons. For that we need to know the probability of occupation,  $F(E)$ . This function can be obtained by a not-too-laborious exercise in statistical mechanics. One starts with the Pauli principle (that no state can be occupied by more than one electron) and works out the most



probable distribution on the condition that the total energy and the total number of particles are given. The result is the so-called Fermi-Dirac distribution

$$F(E) = \frac{1}{\{\exp(E - E_F)/kT\} + 1} \quad (6.11)$$

where  $E_F$  is a parameter called the Fermi level. It has the easily memorized property that at

$$E = E_F, \quad F(E) = \frac{1}{2}, \quad (6.12)$$

that is, at the Fermi level the probability of occupation is  $\frac{1}{2}$ .

As may be seen in Fig. 6.1,  $F(E)$  looks very different from the classical distribution  $\exp(-E/kT)$ . Let us analyse its properties in the following cases:

1. At  $T = 0$ ,

$$\begin{aligned} F(E) &= 1 & \text{for } E < E_F \\ F(E) &= 0 & \text{for } E > E_F. \end{aligned} \quad (6.13)$$

Thus at absolute zero temperature all the available states are occupied up to  $E_F$  and all the states above  $E_F$  are empty. But, remember,  $Z(E)dE$  is the number of states between  $E$  and  $E + dE$ . Thus the total number of states is

$$\int_0^{E_F} Z(E) dE, \quad (6.14)$$

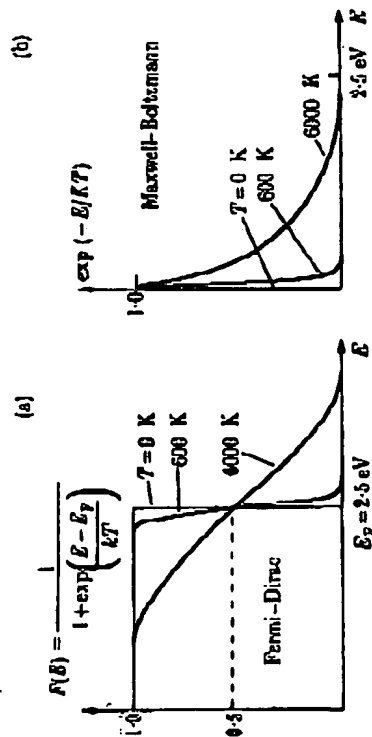


Fig. 6.1 (a) The Fermi-Dirac distribution function for a Fermi energy of 2.5 eV and for temperatures of 0 K, 600 K, and 6000 K. (b) The classical Maxwell-Boltzmann distribution function of energies for the same temperatures

which must equal the total number of electrons  $NL^3$  where  $N$  is the number of electrons per unit volume. Thus substituting eqn (6.10) into eqn (6.14) the following equation must be satisfied:

$$(4\pi L^3/2m)^{3/2}/h^3 \int_0^{E_F} E^{1/2} dE = NL^3. \quad (6.15)$$

Integrating and solving for  $E_F$  we get

$$E_F = \frac{h^2}{2m} \left( \frac{3N}{8\pi} \right)^{2/3}. \quad (6.16)$$

$E_F$ , calculated from the number of free electrons, is shown in Table 6.1. Thus, even at absolute zero temperature, the electrons' energies cover a wide range. This is strongly in contrast with classical statistics where at  $T = 0$  all electrons have zero energy.

2. For electron energies above the Fermi level, so that

$$E - E_F \gg kT, \quad (6.17)$$

the term unity in eqn (6.11) may be neglected, leading to

$$F(E) \approx \exp\{-(E - E_F)/kT\}, \quad (6.18)$$

which you may recognize as the classical Maxwell-Boltzmann distribution. That is, for sufficiently large energies the Fermi-Dirac distribution is reduced to the Maxwell-Boltzmann distribution, generally referred to as the 'Boltzmann tail'.

3. For electron energies below the Fermi level, so that

$$E_F - E \gg kT, \quad (6.19)$$

eqn (6.11) may be approximated by

$$F(E) \approx 1 - \exp(E - E_F)/kT, \quad (6.20)$$

that is, the probability of a state being occupied is very close to unity.

Table 6.1 Fermi levels calculated from eqn (6.16)

Li	4.72 eV
Na	3.12 eV
K	2.14 eV
Rb	1.82 eV
Cs	1.53 eV
Cu	7.04 eV
Ag	5.51 eV
Al	11.70 eV

It is sometimes useful to talk about the probability of a state *not* being occupied and use the function  $1 - F(E)$ . We may say then for the present case that the probability of non-occupation varies exponentially.

4. In the range  $E \approx E_F$  the distribution function changes rather abruptly from nearly unity to nearly zero. The rate of change depends on  $kT$ . For absolute zero temperature the change is infinitely fast, for higher temperatures (as can be seen in Fig. 6.1) it is more gradual. We may take this central region (quite arbitrarily) as between  $F(E) = 0.9$  and  $F(E) = 0.1$ . The width of the region comes out then (by solving eqn (6.11)) to about  $4.4kT$ .

Summarizing, we may distinguish three regions for finite temperatures: from  $E = 0$  to  $E = E_F - 2.2kT$ , where the probability of occupation is close to unity, and the probability of non-occupation varies exponentially; from  $E = E_F - 2.2kT$  to  $E = E_F + 2.2kT$ , where the distribution function changes over from nearly unity to nearly zero; and from  $E = E_F + 2.2kT$  to  $E = \infty$ , where the probability of occupation varies exponentially.

### 6.3 The specific heat of electrons

Classical theory, as I have mentioned before, failed to predict the specific heat of electrons. Now we can see the reason. The real culprit is not the wave nature of the electron nor Schrödinger's equation but Pauli's principle. Since only one electron can occupy a state, electrons of lower energy are not in a position to accept the small amount of energy offered to them occasionally. The states above them are occupied, so they stay where they are. Only the electrons in the vicinity of the Fermi level have any reasonable chance of getting into states of higher energy; so they are the only ones capable of contributing to the specific heat.

The specific heat at constant volume per electron is defined as

$$c_v = \frac{d\langle E \rangle}{dT}, \quad (6.21)$$

where  $\langle E \rangle$  is the average energy of electrons.

A classical electron would have an average energy  $\frac{3}{2}kT$ . On the basis of the Fermi-Dirac distribution we may make a very rough estimate, saying that only the electrons in the region  $E = E_F - 2.2kT$  to  $E = E_F$  get promoted, that is, only a portion  $2.2kT/E_F$  of the electrons behave as classical electrons. Hence the average energy is

$$\langle E \rangle = \frac{3}{2}kT \frac{2.2kT}{E_F}, \quad (6.22)$$

which gives for the specific heat

$$c_v = 6.6 \frac{k^2}{E_F} T. \quad (6.23)$$

A proper derivation of the specific heat would run into mathematical difficulties but it is very simple in principle. The average energy of an electron following a distribution  $F(E)$  is given by

$$\langle E \rangle = \frac{1}{N} \int_0^\infty F(E) Z(E) E dE, \quad (6.24)$$

which should be evaluated as a function of temperature† and differentiated. The result is

$$c_v = \frac{\pi^2 k^2}{2 E_F} T, \quad (6.25)$$

which agrees reasonably well with eqn (6.23) obtained by heuristic arguments. This electronic specific heat is vastly lower than the classical value  $(3/2)k$  for any temperature at which a material can remain solid.

### 6.4 The work function

If the metal is heated or light waves are incident upon it then electrons may leave the metal. A more detailed experimental study would reveal that there is a certain threshold energy the electrons should possess in order to be able to escape. We call this energy (for historical reasons) the *work function* and denote it by  $\phi$ . Thus our model is as shown in Fig. 6.2. At absolute zero temperature all the states are filled up to  $E_F$  and there is an external potential barrier  $\phi$ .

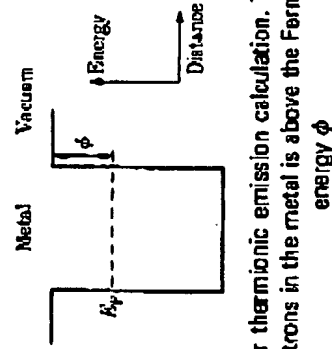


Fig. 6.2 Model for thermionic emission calculation. The potential barrier that keeps the electrons in the metal is above the Fermi energy level by an energy  $\phi$ .

† See, for example, F. Seitz, *Modern theory of solids*, McGraw-Hill, New York, 1940, p. 146.

## 8.2 Intrinsic semiconductors

The aim in semiconductor technology is to purify the material as much as possible and then to introduce impurities in a controlled manner. We shall call the pure semiconductor 'intrinsic' since its behaviour is determined by its intrinsic properties alone, and we shall call the semiconductor 'extrinsic' after external interference has changed its inherent properties. In devices it is mostly extrinsic semiconductors that are used, but it is better to approach our subject gradually and discuss intrinsic semiconductors first.

To be specific, let us think about silicon (though most of our remarks will be qualitatively true of germanium and other semiconductors). Silicon has the diamond crystalline structure; the four covalent bonds are symmetrically arranged. All the four valence electrons of each atom participate in the covalent bonds as we discussed before. But now, having learned band theory, we may express the same fact in a different way. We may say that all the electrons are in the valence band at 0 K. There is an energy gap of 1.1 eV above this before the conduction band starts. Thus to get an electron in a state in which it can take up kinetic energy from an electric field and can contribute to an electric current, we first have to give it a package of at least 1.1 eV of energy. This can come from thermal excitation, or by photon excitation quite independently of temperature.

Let us try to work out now the number of electrons likely to be free to take part in conduction at a temperature  $T$ . How can we do this? We have already solved this problem for the one-dimensional case: eqn (7.51) gives us the effective number of electrons in a partly filled band; so all we need to do is to include the Fermi function to take account of finite temperature and to generalize the whole thing to three dimensions. It can be done but it is a bit too complicated. We shall do something else, which is less justifiable on strictly theoretical grounds but is physically much more attractive. It is really cheating because we use only those concepts of band theory that suit us, and instead of solving the problem honestly we shall appeal to approximations and analogies. It is a compromise solution that will lead us to easily manageable formulae.

First of all we shall say that the only electrons and holes that matter are those near the bottom of the conduction band and the top of the valence band respectively. Thus we may assume that

$$k_x a, k_y b, k_z c \ll 1 \quad (8.1)$$

and we may expand eqn (7.34) to get the energy in the form

$$E = E_1 - 2A_x(1 - \frac{1}{2}k_x^2 a^2) - 2A_y(1 - \frac{1}{2}k_y^2 b^2) - 2A_z(1 - \frac{1}{2}k_z^2 c^2). \quad (8.2)$$

Using our definition of effective mass we can easily show from the above equation that

$$m_x^* = \frac{\hbar^2}{2A_x a^2}, \quad m_y^* = \frac{\hbar^2}{2A_y b^2}, \quad m_z^* = \frac{\hbar^2}{2A_z c^2}. \quad (8.3)$$

Substituting the values of  $A_x a^2$ ,  $A_y b^2$ , and  $A_z c^2$  from eqn (8.3) back into eqn (8.2) and condensing the constant terms into a single symbol,  $E_0$ , we may now express the energy as

$$E = E_0 + \frac{\hbar^2}{2} \left( \frac{k_x^2}{m_x^*} + \frac{k_y^2}{m_y^*} + \frac{k_z^2}{m_z^*} \right). \quad (8.4)$$

Taking further  $E_0 = 0$ , and assuming that everything is symmetric, that is

$$m_x^* = m_y^* = m_z^* = m^*, \quad (8.5)$$

we get

$$E = \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2 + k_z^2). \quad (8.6)$$

This formula is identical to eqn (6.2) obtained from the free electron model. Well, nearly identical. The mass in the denominator is not the real mass of an electron but the effective mass. But that is the only difference. Thus we are going to claim that electrons in the conduction band have a different mass but apart from that behave in the same way as free electrons. Hence the formula derived for the density of states (eqn (6.10)) is also valid and we can use the same method to determine the Fermi level. So we shall have the total number of electrons by integrating.... Wait, we forgot about holes. How do we include them? Well, if holes are the same sort of things as electrons apart from having a positive charge, then everything we said about electrons in the conduction band should be true for holes in the valence band. The only difference is that the density of states must increase downwards for holes. Choosing now the zero of energy at the top of the valence band we may write the density of states in the form

$$Z(E) = C_v (E - E_g)^{1/2}, \quad C_v = 4\pi(2m_v^*)^{3/2}/h^3 \quad (8.7)$$

for electrons, and

$$Z(E) = C_h (-E)^{1/2}, \quad C_h = 4\pi(2m_h^*)^{3/2}/h^3 \quad (8.8)$$

for holes, both of them per unit volume. This is shown in Fig. 8.1, where  $E$  is plotted against  $Z(E)$ . You realize of course that the density of states

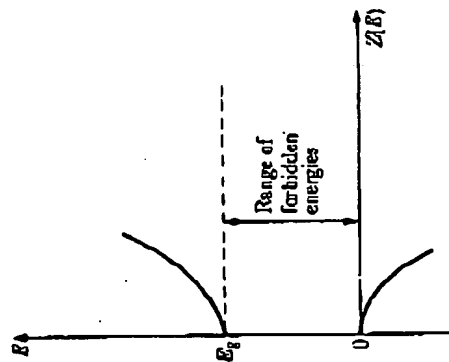


Fig. 8.1 Density of states plotted as a function of energy for the bottom of the conduction (electrons) and top of the valence (holes) bands. See eqns (8.7) and (8.8)

has meaning only in the allowed energy band and must be identically zero in the gap between the two bands.

Let us return now to the total number of electrons. To obtain that we must take the density of states, multiply by the probability of occupation (getting thereby the total number of occupied states), and integrate from the bottom to the top of the conduction band. So, formally, we have to solve the following integral

$$N_c = \int_{\text{bottom of conduction band}}^{\text{top of conduction band}} (\text{density of states})(\text{Fermi function}) dE \quad (8.9)$$

There are several difficulties with this integral:

1. Our solution for the density of states is valid only at the bottom of the band,
2. The Fermi function

$$F(E) = \left\{ 1 + \exp\left(\frac{E - E_F}{kT}\right) \right\}^{-1} \quad (8.10)$$

is not particularly suitable for analytical integration.

3. We would need one more parameter in order to include the width of the conduction band.

We are saved from all these difficulties by the fact that the Fermi level lies in the forbidden band and in practically all cases of interest its distance from the band edge is large in comparison with  $kT$  (0.025 eV at

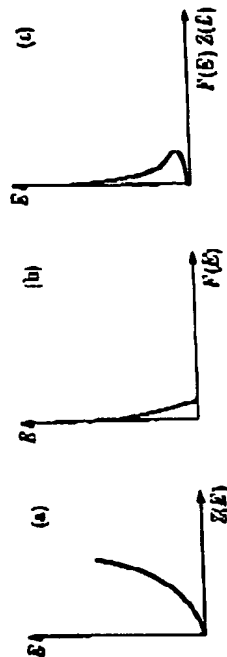


Fig. 8.2 (a) The density of states as a function of energy for the bottom of the conduction band. (b) The Fermi function for the same range of energies. (c) A plot of  $F(E)Z(E)$  showing that the filled electron states are clustered together close to the bottom of the conduction band

room temperature). Hence,

$$E - E_F \gg kT \quad (8.11)$$

and the Fermi function may be approximated by

$$F(E) = \exp\{-(E - E_F)/kT\}. \quad (8.12)$$

as shown already in eqn (6.18).

If the Fermi function declines exponentially, then the  $F(E)Z(E)$  product will be appreciable only near the bottom of the conduction band as shown in Fig. 8.2. Thus we do not need to know the density of states for higher energies (nor the width of the band) because the fast decline of  $F(E)$  will make the integrand practically zero above a certain energy. But if the integrand is zero anyway, why not extend the upper limit up to infinity? We may then come to an integral that is known to mathematicians.

Substituting now equations (8.7) and (8.12) into eqn (8.9) we get

$$N_c = C_c \int_{E_g}^{\infty} (E - E_g)^{1/2} \exp\{-(E - E_F)/kT\} dE. \quad (8.13)$$

Introducing now the new variable

$$x = (E - E_g)/kT \quad (8.14)$$

the integral takes the form

$$N_c = C_c (kT)^{3/2} \exp\{-(E_g - E_F)/kT\} \int_0^{\infty} x^{1/2} e^{-x} dx. \quad (8.15)$$

According to mathematical tables of high reputation†

$$\int_0^{\infty} x^{1/2} e^{-x} dx = \frac{1}{2}\sqrt{\pi} \quad (8.16)$$

† Even better, you could work it out for yourself; it's not too difficult.

leading to the final result

$$N_c = N_c \exp\{-(E_g - E_F)/kT\}, \quad (8.17)$$

where

$$N_c = 2(2\pi m_e^* kT/h^2)^{3/2}. \quad (8.18)$$

Thus we have obtained the number of electrons in the conduction band as a function of some fundamental constants, of temperature, of the effective mass of the electron at the bottom of the band, and of the amount of energy by which the bottom of the band is above the Fermi level.

We can deal with holes in an entirely analogous manner. The probability of a hole being present (that is of an electron being absent) is given by the function

$$1 - F(E), \quad (8.19)$$

which also declines exponentially along the negative  $E$ -axis. So we can choose the lower limit of integration as  $-\infty$ , leading to the result for the number of holes in the valence band:

$$N_h = N_v \exp(-E_F/kT), \quad (8.20)$$

where

$$N_v = 2(2\pi m_h^* kT/h^2)^{3/2}. \quad (8.21)$$

For an intrinsic semiconductor each electron excited into the conduction band leaves a hole behind in the valence band. Therefore the number of electrons should be equal to the number of holes (this would actually follow from the condition of charge neutrality too), that is

$$N_c = N_h. \quad (8.22)$$

Substituting now equations (8.17) and (8.20) into eqn (8.22) we get

$$N_c \exp\{-(E_g - E_F)/kT\} = N_v \exp(-E_F/kT), \quad (8.23)$$

from which the Fermi level can be determined. With a little algebra we get

$$E_F = \frac{E_g}{2} + \frac{1}{2} kT \log_e \frac{m_h^*}{m_e^*}. \quad (8.24)$$

Since  $kT$  is small and the effective masses of electrons and holes are not very much different, we can say that the Fermi level is roughly halfway between the valence and conduction bands.

We know now everything we need about intrinsic semiconductors. Let us now look at the effect of impurities.

### 8.3 Extrinsic semiconductors

We shall continue to consider silicon as our specific example, but now with a controlled addition of a group V impurity (this refers to column 15 in the periodic table of elements), as, for example, antimony (Sb), arsenic (As), or phosphorus (P). If the impurity is less than, say,  $1$  in  $10^6$  silicon atoms, the lattice will be hardly different from that of a pure silicon crystal. Each group V atom will replace a silicon atom and use up four of its valence electrons for covalent bonding (Fig. 8.3a). There will, however, be a spare electron. It will no longer be so tightly bound to its nucleus as in a free group V atom, since the outer shell is now occupied (we might look at it this way) by eight electrons, the inert gas number; so the dangling spare electron cannot be very tightly bound. However, the impurity nucleus still has a net positive charge to distinguish it from its neighbouring silicon atoms. Hence we must suppose that the electron still has some affinity for its parent atom. Let us rephrase this somewhat anorthomorphic picture in terms of the band theory. We have said the energy gap represents the minimum energy required to ionize a silicon atom by taking one of its valency electrons. The electron belonging to the impurity atom clearly needs far less energy than this to become available for conduction. We would expect its energy level to be something like  $E_D$  in Fig. 8.3b. ( $E_g - E_D$ ) is typically of the order of  $10^{-2}$  eV (see Table 8.1). At absolute zero temperature an electron does occupy this energy level, and it is *not* available for conduction. But at finite temperatures this electron needs no more than about  $10^{-2}$  eV of energy to put it into the conduction band. This phenomenon is usually referred to as an

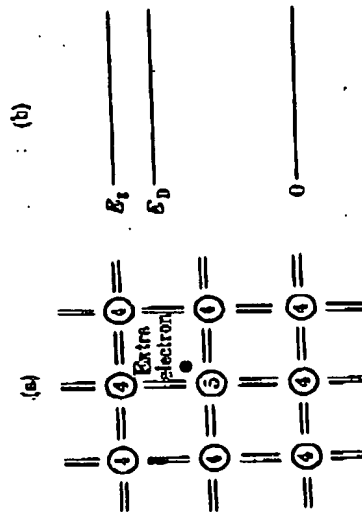


Fig. 8.3 (a) The extra electron 'belonging' to the group V impurity is much more weakly bound to its parent atom than the electrons taking part in the covalent bond. (b) This is equivalent to a donor level close to the conduction band in the band representation